

Exact Solutions in Nonlocal Linear Models

S. Yu. Vernov*

Skobeltsyn Institute of Nuclear Physics, Moscow State University,
Vorobyevy Gory, 119991, Moscow, Russia

Abstract

A general class of cosmological models driven by a nonlocal scalar field inspired by the string field theory is studied. Using the fact that the considering linear nonlocal model is equivalent to an infinite number of local models we have found an exact special solution of the nonlocal Friedmann equations. This solution describes a monotonically increasing Universe with the phantom dark energy.

1 Introduction

Recently string theory has been intensively discussed as promising candidates for the theoretical explanation (see for example [1, 2]) of the obtained experimental data [3].

The purpose of this paper is to present recent results concerning studies of the string field theory (SFT) inspired nonlocal cosmological models in the Friedmann–Robertson–Walker Universe. A Distinguished feature of these models [4]–[14] is the presence of an infinite number of higher derivative terms. For special values of the parameters these models describe linear approximations to the cubic bosonic or nonBPS fermionic SFT nonlocal tachyon models, p-adic string models or the models with the invariance of the action under the shift of the dilaton field to a constant. The NonBPS fermionic SFT nonlocal tachyon model has been considered as a candidate for the dark energy [4]. This report is based on papers [11], which have been made in cooperation with I.Ya. Aref'eva and L.V. Joukovskaya.

2 Nonlocal linear models

Let us consider a model of gravity coupling with a SFT inspired nonlocal scalar field

$$S = \int d^4x \sqrt{-g} \left(\frac{m_p^2}{2} R + \frac{\xi^2}{2} \phi \square_g \phi + \frac{1}{2} (\phi^2 - c \Phi^2) - \Lambda' \right), \quad (1)$$

where $\Phi = e^{\square_g \phi}$, $g_{\mu\nu}$ is the metric tensor (the signature is $(-, +, +, +)$), $\square_g = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$, $m_p^2 = g_4 M_p^2 / M_s^2$, M_p is a mass Planck, M_s is a characteristic string scale related with the string tension α' : $M_s = 1/\sqrt{\alpha'}$, ϕ is a dimensionless scalar field and g_4 is a dimensionless four dimensional effective coupling constant. An effective four dimensional cosmological constant is $\Lambda = \frac{M_s^4}{g_4} \Lambda'$. Parameters ξ and c are positive. The term $(e^{\square_g \phi})^2$ is analogous to the interaction term for the tachyon in the string action.

* E-mail: svernov@theory.sinp.msu.ru

We consider the case of the spatially flat Friedmann–Robertson–Walker Universe:

$$ds^2 = -dt^2 + a^2(t) (dx_1^2 + dx_2^2 + dx_3^2) \quad (2)$$

and spatially homogeneous solutions $\phi(t)$. In this case $T_{\alpha\beta} = g_{\alpha\beta} \text{diag}\{\mathcal{E}, -\mathcal{P}, -\mathcal{P}, -\mathcal{P}\}$, where the energy density \mathcal{E} and pressure \mathcal{P} are as follows

$$\mathcal{E} = \mathcal{E}_k + \mathcal{E}_{nl2} + \mathcal{E}_p + \mathcal{E}_{nl1} + \Lambda', \quad \mathcal{P} = \mathcal{E}_k + \mathcal{E}_{nl2} - \mathcal{E}_p - \mathcal{E}_{nl1} - \Lambda', \quad (3)$$

$$\begin{aligned} \mathcal{E}_k &= \frac{\xi^2}{2} (\partial_0 \phi)^2, & \mathcal{E}_{nl2} &= -c \int_0^1 (\partial_0 e^{(1+\rho)\mathcal{D}} \phi) (\partial_0 e^{(1-\rho)\mathcal{D}} \phi) d\rho, \\ \mathcal{E}_p &= -\frac{1}{2} (\phi^2 - c (e^{\mathcal{D}} \phi)^2), & \mathcal{E}_{nl1} &= c \int_0^1 (e^{(1+\rho)\mathcal{D}} \phi) (-\mathcal{D} e^{(1-\rho)\mathcal{D}} \phi) d\rho, \end{aligned} \quad (4)$$

where $\mathcal{D} \equiv -\partial_0^2 - 3H(t)\partial_0$, the Hubble parameter $H \equiv \dot{a}/a$ and dot denotes the time derivative ($\dot{a} \equiv \partial_0 a$). Note that the energy-momentum tensor $T_{\alpha\beta}$ includes the nonlocal terms, so the Einstein's equations

$$3H^2 = \frac{1}{m_p^2} \mathcal{E}, \quad \dot{H} = -\frac{1}{2m_p^2} (\mathcal{E} + \mathcal{P}) \quad (5)$$

are nonlocal ones. The second equation of (5) is the nonlinear integral equation in $H(t)$:

$$\dot{H} = -\frac{1}{m_p^2} \left(\frac{\xi^2}{2} (\partial_0 \phi)^2 - c \int_0^1 (\partial_0 e^{(1+\rho)\mathcal{D}} \phi) (\partial_0 e^{(1-\rho)\mathcal{D}} \phi) d\rho \right). \quad (6)$$

3 The equation of motion

In the spatially flat Friedmann–Robertson–Walker Universe we get the following equation of motion for the space homogeneous scalar field ϕ

$$(\xi^2 \mathcal{D} + 1) e^{-2\mathcal{D}} \phi = c \phi. \quad (7)$$

Really this equation is a consequence of the Einstein's equations, hence, both the metric $g_{\mu\nu}$ and the scalar field ϕ are unknown. We assume that the metric $g_{\mu\nu}$ is given and consider eq. (7) as an equation in ϕ . Solutions of the following linear differential equation

$$-\mathcal{D}\phi \equiv \ddot{\phi}(t) + 3H(t)\dot{\phi}(t) = \alpha^2 \phi, \quad (8)$$

represent the solutions of eq. (7) with α , which is a root of the characteristic equation

$$F(\alpha^2) \equiv -\xi^2 \alpha^2 + 1 - c e^{-2\alpha^2} = 0. \quad (9)$$

Equation (9) has the following roots

$$\alpha_n = \pm \frac{1}{2\xi} \sqrt{4 + 2\xi^2 W_n \left(-\frac{2c e^{-2/\xi^2}}{\xi^2} \right)}, \quad n = 0, \pm 1, \pm 2, \dots, \quad (10)$$

where W_n is the n -s branch of the Lambert function, satisfying a relation $W(z)e^{W(z)} = z$. The Lambert function is a multivalued function, so eq. (9) has an infinite number of roots. Parameters ξ and c are real, therefore if α_n is a root of (9), then the adjoined number α_n^* is a root as well. Note that if α_n is a root of (9), then $-\alpha_n$ is a root as well.

If $\alpha^2 = \alpha_0^2$ is a multiple root, then at this point $F(\alpha_0^2) = 0$ and $F'(\alpha_0^2) = 0$. Double roots exist if and only if

$$c = \frac{\xi^2}{2} e^{(2/\xi^2 - 1)}. \quad (11)$$

Note that existence of double roots means that there exist solutions of equation (7), which do not satisfy to equation (8), but satisfy the following equation $\square_g^2 \phi = \alpha^4 \phi$. In the flat case an example of a such solution is the function $\phi(t) = t \exp(\alpha t)$ (see[11]). All roots for any ξ and c are no more than double degenerated, because $F''(\alpha_0^2) \neq 0$. We consider such values of ξ and c that equality (11) is not satisfied and all roots are single.

Let us make an assumption, that $\phi_1(t)$ and $H_1(t)$ satisfy eq. (8), with $\alpha = \alpha_1$ is a root of eq. (9), hence, eq. (7) is solved. The energy density and the pressure are as follows:

$$E(\phi_1) = \frac{\eta_{\alpha_1}}{2} ((\partial_0 \phi_1)^2 - \alpha_1^2 \phi_1^2), \quad P(\phi_1) = \frac{\eta_{\alpha_1}}{2} ((\partial_0 \phi_1)^2 + \alpha_1^2 \phi_1^2), \quad \eta_{\alpha_1} \equiv \xi^2 + 2\xi^2 \alpha_1^2 - 2.$$

Using this formula, we rewrite system (5) in the following form:

$$3H_1^2 = \frac{\eta_{\alpha_1}}{2m_p^2} (\dot{\phi}_1^2 - \alpha_1^2 \phi_1^2 + \Lambda'), \quad \dot{H}_1 = -\frac{\eta_{\alpha_1}}{2m_p^2} \dot{\phi}_1^2. \quad (12)$$

So, our assumption allows to transform a system with a nonlocal scalar field into a system with a local scalar field ϕ_1 . Note that in such a way we obtain an infinity number of local systems, because eq. (9) has an infinity number of roots.

4 Exact Solutions

In this paper we present an exact solution, which looks realistic for the SFT inspired cosmological model. One of the possible scenarios of the Universe evolution considers the Universe to be a D3-brane (3 spatial and 1 time variable) embedded in higher-dimensional space-time. The D-brane is unstable and does evolve to the stable state. This process is described by the open string dynamics, which ends are attached to the brane. A phantom scalar field is an open string tachyon. According to the Sen's conjecture [15], the tachyon describes brane decay, at which a slow transition to the stable vacuum, correlating with only states of the closed string, takes place. This picture allows us to specify the asymptotic conditions for the phantom field $\phi(t)$. We assume that $\phi(t)$ smoothly rolls from the unstable perturbative vacuum ($\phi = 0$) to a nonperturbative one, for example $\phi = A_0$, where A_0 is a nonzero constant, and stops there. In other words we seek a king-like solution.

At $c = 1$ one of solutions of eq. (9) is $\alpha = 0$ and at $\Lambda' > 0$ we obtain a real solution

$$H_1(t) = \sqrt{\frac{\Lambda'}{6m_p^2}} \tanh(\tilde{t}), \quad \phi_1(t) = \pm \sqrt{\frac{2m_p^2}{3(2 - \xi^2)}} \arctan(\sinh(\tilde{t})) + C_1, \quad (13)$$

where $\tilde{t} \equiv \sqrt{\frac{3\Lambda'}{2m_p^2}}(t - t_0)$, t_0 and C_1 are arbitrary real constants. We have obtained that $\phi(t)$ can be a real scalar field if and only if it is a phantom one ($\eta_\alpha < 0$, that is equivalent to $\xi^2 < 2$). We have obtained the general solution for (12). The Hubble parameter $H(t)$ is a monotonically increasing function, so solution (13) corresponds to phantom dark energy.

5 Conclusions

We have studied the SFT inspired linear nonlocal model. This model has an infinite number of higher derivative terms and are characterized by two positive parameters. For some values of these parameters the corresponding actions are linear approximations to either the bosonic or nonBPS fermionic cubic SFT as well as to the nonpolynomial SFT.

We have shown that our linear model with one nonlocal scalar field generates an infinite number of local models. Some of these models can be solved explicitly and, hence, special exact solutions for nonlocal model in the Friedmann metric can be obtained [11, 12]. We have constructed an exact kink-like solution, which correspond to monotonically increasing Universe with phantom dark energy. Note that the obtained behaviour of the Hubble parameter is close to behavior of the Hubble parameter in the nonlinear nonlocal model [4], which recently has been obtained numerically [14]. Note that the considering nonlocal model generates local models with an arbitrary number of local scalar fields as well [11].

Author is grateful to I.Ya. Aref'eva and L.V. Joukovskaya for the collaboration and useful discussions. Author is thankful to the organizers of the International Workshop on Supersymmetries and Quantum Symmetries ("SQS'07") for hospitality and financial support. This research is supported in part by RFBR grant 05-01-00758 and by Russian President's grant NSh-8122.2006.2.

References

- [1] L. McAllister and E. Silverstein, arXiv:0710.2951
- [2] T. Biswas, A. Mazumdar and W. Siegel, *JCAP* **0603** (2006) 009, hep-th/0508194; R. Lazkoz, R. Maartens and E. Majerotto, *Phys. Rev.* **D74** (2006) 083510, astro-ph/0605701; Sh. Nojiri and S.D. Odintsov, *Phys. Rev.* **D74** (2006) 046004, hep-th/0605039
- [3] D.N. Spergel et al., *Astroph. J. Suppl. Ser.* **170** (2007) 377–408, astro-ph/0603449
- [4] I.Ya. Aref'eva, *AIP Conf. Proc.* **826** (2006) 301–311, astro-ph/0410443; I.Ya. Aref'eva, *AIP Conf. Proc.* **957** 297 arXiv:0710.3017
- [5] I.Ya. Aref'eva and L.V. Joukovskaya, *JHEP* **0510** (2005) 087, hep-th/0504200
- [6] G. Calcagni, *JHEP* **0605** (2006) 012, hep-th/0512259, G. Calcagni, M. Montobbio and G. Nardelli, *Phys. Rev.* **D 76** (2007) 126001, arXiv:0705.3043
- [7] I.Ya. Aref'eva and A.S. Koshelev, *JHEP* **0702** (2007) 041, hep-th/0605085
- [8] I.Ya. Aref'eva and I.V. Volovich, hep-th/0612098.
- [9] N. Barnaby, T. Biswas and J.M. Cline, *JHEP* **0704** (2007) 056, hep-th/0612230; N. Barnaby and J.M. Cline, *JCAP* **0707** (2007) 017, arXiv:0704.3426; N. Barnaby and N. Kamran, *JHEP* **0802** (2008) 008, arXiv:0709.3968
- [10] A.S. Koshelev, *JHEP* **0704** (2007) 029, hep-th/0701103,
- [11] I.Ya. Aref'eva, L.V. Joukovskaya and S.Yu. Vernov, *JHEP* **0707** (2007) 087, hep-th/0701184; I.Ya. Aref'eva, L.V. Joukovskaya, and S.Yu. Vernov, arXiv:0711.1364, to be published in *J. Phys. A: Math. Theor.*
- [12] I.Ya. Aref'eva and I.V. Volovich, *Int. J. Geom. Meth. Mod. Phys.* **4** (2007) 881–895, hep-th/0701284
- [13] J.E. Lidsey, *Phys. Rev.* **D76** (2007) 043511, hep-th/0703007
- [14] L.V. Joukovskaya, *Phys. Rev.* **D76** (2007) 105007, arXiv:0707.1545; L.V. Joukovskaya, *AIP Conf. Proc.* **957** 325, arXiv:0710.0404
- [15] A. Sen, *Int. J. Mod. Phys. A* **20** (2005) 5513–5656, hep-th/0410103